STEP II, 2004, Q4 MS

Q4 The first two parts of this question depend on the identity $\tan^{-1} A + \tan^{-1} B = \tan^{-1}[(A + B)/(1 - AB)]$ which is simply another way of writing $\tan(\alpha+\beta) = [\tan\alpha + \tan\beta]/[1 - \tan\alpha\tan\beta]$.

In the second part, it follows from the data that

$$\tan^{-1}[1/(p+q+s)] + \tan^{-1}[1/(p+q+t)] = \tan^{-1}[1/(p+q)]$$

and

$$\tan^{-1}[1/(p+r+u)] + \tan^{-1}[1/(p+r+v)] = \tan^{-1}[1/(p+r)].$$

As also from the data,

$$\tan^{-1}[1/(p+q)] + \tan^{-1}[1/(p+r)] = \tan^{-1}(1/p),$$

then the proof is complete.

For the final part, it is clear that $p = 7$ and this leads to $st = (7+q)^2 + 1$, $uv = (7+r)^2$, $qr = 50$. From the second displayed result it is obvious that $q + s = 6$, $q + t = 14$, so that $(7+q)^2 + 1 = (6-q)(14-q) \Rightarrow q = 1$, and hence $s = 5$, $t = 13$. The values $r = 50$, $u = 25$, $v = 130$ can be obtained by a similar strategy.

The solution given above is not unique. Moreover, other plausible strategies may lead to incorrect solutions. It is important, therefore, to check that the solution obtained not only satisfies the displayed identity, but also the given conditions.

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