

STEP II, 2004, Q4

- 4 The positive numbers a , b and c satisfy $bc = a^2 + 1$. Prove that

$$\arctan\left(\frac{1}{a+b}\right) + \arctan\left(\frac{1}{a+c}\right) = \arctan\left(\frac{1}{a}\right).$$

The positive numbers p , q , r , s , t , u and v satisfy

$$st = (p+q)^2 + 1, \quad uv = (p+r)^2 + 1, \quad qr = p^2 + 1.$$

Prove that

$$\arctan\left(\frac{1}{p+q+s}\right) + \arctan\left(\frac{1}{p+q+t}\right) + \arctan\left(\frac{1}{p+r+u}\right) + \arctan\left(\frac{1}{p+r+v}\right) = \arctan\left(\frac{1}{p}\right).$$

Hence show that

$$\arctan\left(\frac{1}{13}\right) + \arctan\left(\frac{1}{21}\right) + \arctan\left(\frac{1}{82}\right) + \arctan\left(\frac{1}{187}\right) = \arctan\left(\frac{1}{7}\right).$$

[Note that $\arctan x$ is another notation for $\tan^{-1} x$.]



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