STEP II, 2005, Q3 EC

Q3 This very popular question provided an opportunity for candidates to show their competence with basic A-level mathematics. Only the last part was at all unusual. Unexpectedly, most responses showed at least one error in (ii), but contained sound mathematics to establish the final inequality.

For the introductory inequality it is only necessary to state that $y(0) = 0$, $y(\pi/2) = 1$ and to show that $dy/dx = 0$ at the origin and that $dy/dx > 0$ for $0 < x \leq \pi/2$. The majority of responses proceeded in this way, though the layout of the working was unclear in some instances. The majority of sketch graphs were incorrect and/or incomplete. Common errors were a non-zero gradient at the origin and/or a zero gradient at the point $(\pi/2, 1)$.

(i) Most responses showed accurate and complete working to establish the displayed result.

(ii) The working in almost all responses soon led to the preliminary result that $J = \int_0^{\pi/2} y^2\, dx = I_1 - I_2 + I_3$, where $I_1 = \int_0^{\pi/2} \sin^2 x\, dx$, $I_2 = \int_0^{\pi/2} x \sin 2x\, dx$, $I_3 = \int_0^{\pi/2} x^2 \cos^2 x\, dx$.

The results $I_1 = \pi/4$, $I_2 = \pi/4$ were usually in evidence, though sometimes these followed from erroneous working. In contrast, many candidates were unable to supply the extensive technical detail needed for the determination of I_3. Thus a complete and correct evaluation of J appeared in only a minority of responses.