

## STEP II, 2005, Q2 MS

Q2 (a) (i) Following the definition of  $f(N)$ , it is immediate that  $f(12) = 12(1 - 1/2)(1 - 1/3) = 4$ , and  $f(180) = 180(1 - 1/2)(1 - 1/3)(1 - 1/5) = 48$ .

(ii) The result may seem obvious but care must be taken in order to construct a complete proof. For example,  $N = p_1^{\alpha_1} \dots p_k^{\alpha_k} \Rightarrow f(N) = p_1^{\alpha_1 - 1} \dots p_k^{\alpha_k - 1} (p_1 - 1) \dots (p_k - 1)$ . Thus as  $p_i$  is a positive integer and  $\alpha_i - 1$  is a non-negative integer for  $1 \leq i \leq k$ , then  $f(N)$  is an integer.

(b) In each of (i),(ii),(iii), the conclusion must be made clear.

(i) As  $f(3)f(9) = 2 \times 6 = 12 \neq f(27) = 18$ , then the statement is false.

(ii) For any two primes  $p$  and  $q$ ,  $f(p)f(q) = p(1 - 1/p)q(1 - 1/q) = pq(1 - 1/p)(1 - 1/q) = f(pq)$ . Hence the statement is true.

(iii) Consider  $f(5) = 4$ ,  $f(6) = 2$ ,  $f(30) = 8 = 2 \times 4$ . Then as 6 is not a prime it is clear that the statement is false.

(c) Start with  $p^{m-1}(p-1) = 146410$ , then without difficulty it will be found that  $p = 11$  and  $m = 5$  (not 4).



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