STEP II, 2005, Q2 EC

Q2 This question was also popular and most responses made significant progress with the majority of the sections available.

(a) (i) The correct results for $f(12)$ and $f(180)$ appeared in almost all responses.

(ii) The working here was often hazy and protracted. Notational confusions led to poor reasoning. What was required was an argument based on

$$N = p_1^{\alpha_1} \dots p_k^{\alpha_k} \Rightarrow \dots \Rightarrow f(N) = p_1^{\alpha_1 - 1} \dots p_k^{\alpha_k - 1}(p_1 - 1) \dots (p_k - 1).0$$

(b) In all three parts of this section, it was expected that, at least, the conclusion would be made clear. However, there were many instances where this did not happen.

(i) Most responses showed a suitable counterexample, e.g., $f(3)f(9) = 2 \times 6 = 12 \neq f(27) = 18$, and thus proved that the displayed result lacks generality.

(ii) There were few failures here. The correct conclusion was usually supported by a simple argument such as $f(p)f(q) = p(1 - 1/p)q(1 - 1/q) = pq(1 - 1/p)(1 - 1/q) = f(pq)$.

(iii) Most responses groped there way through working to show, e.g., $f(5) = 4$, $f(6) = 2$, $f(30) = 2 \times 4 = 8$, from which the required conclusion can be made. However, some candidates were unable to do this. In this respect there were erroneous statements of the form $(P \Rightarrow Q) \Rightarrow (Q \Rightarrow P)$ and especially, $(P' \Rightarrow Q') \Rightarrow (P \Rightarrow Q)$.

(c) Responses generally started with $p^{m-1}(p - 1) = 146410$, but some led on to incorrect results, or simply faded out. A popular incorrect conclusion was $p = 11$, $m = 4$.

(a) (i) $f(12) = 4$, $f(180) = 48$: (b) (i) not always true, (ii) true, (iii) false: (c) $p = 11$, $m = 5$.