

## STEP II, 2005, Q2

- 2 For any positive integer  $N$ , the function  $f(N)$  is defined by

$$f(N) = N \left(1 - \frac{1}{p_1}\right) \left(1 - \frac{1}{p_2}\right) \cdots \left(1 - \frac{1}{p_k}\right)$$

where  $p_1, p_2, \dots, p_k$  are the only prime numbers that are factors of  $N$ .  
Thus  $f(80) = 80\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{5}\right)$ .

- (a) (i) Evaluate  $f(12)$  and  $f(180)$ .  
(ii) Show that  $f(N)$  is an integer for all  $N$ .
- (b) Prove, or disprove by means of a counterexample, each of the following:  
(i)  $f(m)f(n) = f(mn)$ ;  
(ii)  $f(p)f(q) = f(pq)$  if  $p$  and  $q$  are distinct prime numbers;  
(iii)  $f(p)f(q) = f(pq)$  only if  $p$  and  $q$  are distinct prime numbers.
- (c) Find a positive integer  $m$  and a prime number  $p$  such that  $f(p^m) = 146410$ .



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