STEP II, 2005, Q1 EC

Q1 This was the most popular question of the paper and the majority of candidates made some progress with it. Many responses were undermined by elementary errors.

At the outset, the preliminary result $d[x^2e^{-x^2}]/dx = 2xe^{-x^2} - 2x^3e^{-x^2}$ appeared in almost all responses. Remarkably, however, not all candidates were able to identify all the roots of $x - x^3 = 0$.

A small minority of candidates thought that $P(x) \equiv x(x^2 - a^2)(x^2 - b^2)$ whereas the correct starting point is $P'(x) - 2xP(x) \equiv x(x^2 - a^2)(x^2 - b^2)$ $(*)$. Actually, some wrote $x(x^2 - a)(x^2 - b)$ for $x(x^2 - a^2)(x^2 - b^2)$.

Beyond this beginning, the best candidates could see immediately that $P(x)$ can take the form $-x^4/2 + px^2 + q$ and this approach certainly simplified later working. In contrast, some started with $P(x) \equiv \sum_{i=0}^{4} c_i x^i$, but this more complicated strategy generated many erroneous solutions.

There were also those who attempted to obtain $P(x)$ from $e^{x^2} \int x(x^2 - a^2)(x^2 - b^2)e^{-x^2}\,dx$. The systematic and correct use of the integration by parts rule will readily show how for $I_n = \int x^{2n+1} e^{-x^2}\,dx$, I_1 and I_2 relate to $I_0 = -(1/2)e^{-x^2}$ in a simple way. In this context, few responses were at all clear and generally notational confusions led to inaccuracy.

$x = -1, 0, 1$: $P(x) =$ any non-zero scaling of $-x^4/2 + (a^2/2 + b^2/2 - 1)x^2 + -1 + a^2/2 + b^2/2 - a^2b^2/2$

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