STEP II, 2005, Q14 MS

Q14 Standard methods lead to $k = 1/(1+\lambda)$ and $\mu = \lambda^2/[2(1+\lambda]$. respectively.

Also $E(X^2) = k + k\lambda^3/3 = (\lambda^3+3)/[3(1+\lambda)]$, so that $\sigma^2 = (\lambda^3+3)/[3(1+\lambda)] - \lambda^4/[4(1+\lambda)^2)] \Rightarrow$ ... $\Rightarrow$ required result.

For the remainder of the question $k = 1/3$.

(i) The graph is made up of three segments corresponding to $x < 0$, $0 \leq x \leq 2$ and $x > 2$. In particular, the middle segment is a translation of the curve $y = \phi(x)/3$ upwards through $1/3$ parallel to the $f(x)$ axis.

(ii) If $F(x)$ is the CDF of X, then

$F(x) = \Phi(x)/3$ for $x \leq 0$,

$F(x) = \Phi(x)/3 + x/3$ for $0 < x \leq 2$,

$F(x) = \Phi(x)/3 + 2/3$ for $x > 2$.

(iii) Begin with $\mu = 2/3$, $\sigma^2 = 7/9$. then from the information given it is clear that

$P(0 < X < \mu + 2\sigma) = .9921/3 + 2/3 - 1/6 = 0.8307$.