

## STEP II, 2005, Q14

- 14 The probability density function  $f(x)$  of the random variable  $X$  is given by

$$f(x) = k [\phi(x) + \lambda g(x)],$$

where  $\phi(x)$  is the probability density function of a normal variate with mean 0 and variance 1,  $\lambda$  is a positive constant, and  $g(x)$  is a probability density function defined by

$$g(x) = \begin{cases} 1/\lambda & \text{for } 0 \leq x \leq \lambda; \\ 0 & \text{otherwise.} \end{cases}$$

Find  $\mu$ , the mean of  $X$ , in terms of  $\lambda$ , and prove that  $\sigma$ , the standard deviation of  $X$ , satisfies.

$$\sigma^2 = \frac{\lambda^4 + 4\lambda^3 + 12\lambda + 12}{12(1 + \lambda)^2}.$$

In the case  $\lambda = 2$ :

- (i) draw a sketch of the curve  $y = f(x)$ ;
- (ii) express the cumulative distribution function of  $X$  in terms of  $\Phi(x)$ , the cumulative distribution function corresponding to  $\phi(x)$ ;
- (iii) evaluate  $P(0 < X < \mu + 2\sigma)$ , given that  $\Phi(\frac{2}{3} + \frac{2}{3}\sqrt{7}) = 0.9921$ .



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