STEP II, 2005, Q13 MS

Q13 The introductory result at the end of the first paragraph is standard. For the approximations exhibited in (i),(ii) and (iii) it is important to ensure that enough terms are taken in the relevant expansions.

(i) $q = 1 - (1+\lambda)e^{-\lambda} = 1 - (1+\lambda)[1 - \lambda + \lambda^2/2 + O(\lambda^3)]$, as $\lambda \to 0$.

Thus $q = \lambda^2/2 + O(\lambda^3) \approx \lambda^2/2$, as $\lambda \to 0$.

(ii) $P(Y = n) = p^n > 1 - \lambda \Rightarrow e^{-n\lambda}(1+\lambda)^n > 1 - \lambda$

$\Rightarrow [1 - n\lambda + n^2\lambda^2/2][1 + n\lambda + n(n-1)\lambda^2/2] + O(\lambda^3) > 1 - \lambda$

which leads to $n < 2/\lambda$ within $O(\lambda^3)$

(iii) Write $P(Y > 1|Y > 0) = P(Y > 1)/P(Y > 0)$

$= (1 - q^n - npq^{n-1})/(1 - q^n) \approx 1 - np(\lambda^2/2)^{n-1}/[1 - (\lambda^2/2)^n]$,

from which follows the required result.