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13 The number of printing errors on any page of a large book of N pages is modelled by a Poisson variate with parameter λ and is statistically independent of the number of printing errors on any other page. The number of pages in a random sample of n pages (where n is much smaller than N and $n \geqslant 2$) which contain fewer than two errors is denoted by Y. Show that $\mathrm{P}(Y = k) = \binom{n}{k} p^k q^{n-k}$ where $p = (1+\lambda)e^{-\lambda}$ and $q = 1 - p$.

Show also that, if λ is sufficiently small,

(i) $q \approx \frac{1}{2}\lambda^2$;

(ii) the largest value of n for which $\mathrm{P}(Y = n) \geqslant 1 - \lambda$ is approximately $2/\lambda$;

(iii) $\mathrm{P}(Y > 1 \mid Y > 0) \approx 1 - n(\lambda^2/2)^{n-1}$.