

## STEP II, 2005, Q13

- 13** The number of printing errors on any page of a large book of  $N$  pages is modelled by a Poisson variate with parameter  $\lambda$  and is statistically independent of the number of printing errors on any other page. The number of pages in a random sample of  $n$  pages (where  $n$  is much smaller than  $N$  and  $n \geq 2$ ) which contain fewer than two errors is denoted by  $Y$ . Show that  $P(Y = k) = \binom{n}{k} p^k q^{n-k}$  where  $p = (1 + \lambda)e^{-\lambda}$  and  $q = 1 - p$ .

Show also that, if  $\lambda$  is sufficiently small,

- (i)  $q \approx \frac{1}{2}\lambda^2$ ;
- (ii) the largest value of  $n$  for which  $P(Y = n) \geq 1 - \lambda$  is approximately  $2/\lambda$ ;
- (iii)  $P(Y > 1 \mid Y > 0) \approx 1 - n(\lambda^2/2)^{n-1}$ .



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