

## **STEP II, 2005, Q12**

- 12 The twins Anna and Bella share a computer and never sign their e-mails. When I e-mail them, only the twin currently online responds. The probability that it is Anna who is online is  $p$  and she answers each question I ask her truthfully with probability  $a$ , independently of all her other answers, even if a question is repeated. The probability that it is Bella who is online is  $q$ , where  $q = 1 - p$ , and she answers each question truthfully with probability  $b$ , independently of all her other answers, even if a question is repeated.
- (i) I send the twins the e-mail: ‘Toss a fair coin and answer the following question. Did the coin come down heads?’. I receive the answer ‘yes’. Show that the probability that the coin did come down heads is  $\frac{1}{2}$  if and only if  $2(ap + bq) = 1$ .
- (ii) I send the twins the e-mail: ‘Toss a fair coin and answer the following question. Did the coin come down heads?’. I receive the answer ‘yes’. I then send the e-mail: ‘Did the coin come down heads?’ and I receive the answer ‘no’. Show that the probability (taking into account these answers) that the coin did come down heads is  $\frac{1}{2}$ .
- (iii) I send the twins the e-mail: ‘Toss a fair coin and answer the following question. Did the coin come down heads?’. I receive the answer ‘yes’. I then send the e-mail: ‘Did the coin come down heads?’ and I receive the answer ‘yes’. Show that, if  $2(ap + bq) = 1$ , the probability (taking into account these answers) that the coin did come down heads is  $\frac{1}{2}$ .



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