# STEP II, 2004, Q8 MS

*Q8*(i) Integration leads to the general solution $t = A - \ln(1-x)$ and $x(0) = 0 \Rightarrow A = 0$. Thus $x = 1 - e^{-t}$.

(ii) Obviously, $(1-x)^{1/2} < (1+x)^{1/2}$ for all $x \in (0,1]$. Hence multiplying this inequality through by $(1-x)^{1/2}$ leads immediately to the required result.

Arguments which go in the wrong direction, e.g., $1 - x < (1-x^2)^{1/2} \Rightarrow \ldots \Rightarrow x - x^2 > 0$, etc., are invalid. It may be possible to salvage them by replacing $' \Rightarrow '$ by $' \Leftarrow '$.

In the case $n = 2$, the substitution $x = \sin y$ will lead to $t = y + B$ and hence to $t = \sin^{-1}(x) + B$ as the general solution. In particular, $x(0) = 0 \Rightarrow B = 0 \Rightarrow x = \sin t$.

Note that the question does not allow the use of the standard form $\int (1-x^2)^{-1/2}\,dx = \sin^{-1}(x) +$ an arbitrary constant, without proof.

(iii) If $G_n$ is the graph of $x$ for $0 \leq x \leq 1$, then the given inequality shows that the gradient of $G_3$ is greater than the gradient of $G_2$ for each $x$ in this interval. (The inequality of (ii) shows that the same is true of $G_2$ in relation to $G_1$.) These considerations will help to clarify ideas when drawing sketches of $G_n$ for $n = 1, 2, 3$ in the same diagram. In particular, the sketch of $G_3$ should make it clear that once $x$ reaches the value 1 it remains there.