

STEP II, 2004, Q7

- 7 The function f is defined by

$$f(x) = 2 \sin x - x.$$

Show graphically that the equation $f(x) = 0$ has exactly one root in the interval $[\frac{1}{2}\pi, \pi]$. This interval is denoted I_0 .

In order to determine the root, a sequence of intervals I_1, I_2, \dots is generated in the following way. If the interval $I_n = [a_n, b_n]$, and $c_n = (a_n + b_n)/2$, then

$$I_{n+1} = \begin{cases} [a_n, c_n] & \text{if } f(a_n)f(c_n) < 0; \\ [c_n, b_n] & \text{if } f(c_n)f(b_n) < 0. \end{cases}$$

By using the approximations $\frac{1}{\sqrt{2}} \approx 0.7$ and $\pi \approx \sqrt{10}$, show that $I_2 = [\frac{1}{2}\pi, \frac{5}{8}\pi]$ and find I_3 .



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