

## STEP II, 2004, Q5 MS

Q5 Using the integration by parts rule it is easy to establish the results  $\int_0^\pi x \sin x \, dx = \pi$  and  $\int_0^\pi x \cos x \, dx = -2$ .

- Write  $\sin(x+t) = \sin x \cos t + \sin t \cos x$  and the result  $f(t) = t + A \sin t + B \cos t$ , where  $A$  and  $B$  are as defined in the question, follows immediately.

- Hence write  $t + A \sin t + B \cos t = t + \int_0^\pi (x + A \sin x + B \cos x) \sin(x+t) \, dx$  (\*\*\*) so that as

$$\int_0^\pi x \sin(x+t) \, dx = \dots = \pi \cos t - 2 \sin t,$$

$$\int_0^\pi \sin x \sin(x+t) \, dx = \dots = (\pi/2) \cos t,$$

$$\int_0^\pi \cos x \sin(x+t) \, dx = \dots = (\pi/2) \sin t,$$

then, by considering the coefficients of  $\cos t$  and  $\sin t$  on both sides of (\*\*\*), it follows that

$$A = -2 + (\pi/2)B, \quad B = \pi + (\pi/2)A \Rightarrow A = -2, \quad B = 0.$$

Alternatively, equations for  $A$  and  $B$  can be obtained by putting  $t = 0$  and  $t = \pi/2$  in (\*\*\*)



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