STEP II, 2004, Q5

5 Evaluate $\int_0^\pi x \sin x \, \mathrm{d}x$ and $\int_0^\pi x \cos x \, \mathrm{d}x$.

The function f satisfies the equation

$$\mathrm{f}(t) = t + \int_0^\pi \mathrm{f}(x) \sin(x + t) \, \mathrm{d}x \, . \qquad (*)$$

Show that

$$\mathrm{f}(t) = t + A \sin t + B \cos t \, , \qquad (**)$$

where $A = \int_0^\pi \mathrm{f}(x) \cos x \, \mathrm{d}x$ and $B = \int_0^\pi \mathrm{f}(x) \sin x \, \mathrm{d}x$.

Use the expression $(**)$ to find A and B by substituting for $\mathrm{f}(t)$ and $\mathrm{f}(x)$ in $(*)$ and equating coefficients of $\sin t$ and $\cos t$.