STEP II, 2004, Q3 MS

Q3 The obtaining of dy/dx in the form required is a routine exercise in differentiation followed by some algebra.

Setting $dy/dx = 0$ shows that there are stationary points where $x = -2/3, 1/2, 2$. Moreover $d^2y/dx^2 = (x-2)^3(12x+1)$ + a term which is necessarily zero when $x = -2/3, 1/2, 2$. Thus d^2y/dx^2 is positive when $x = -2/3$ and negative when $x = 1/2$, so that C has a minimum at $(-2/3, -8192/729)$ and a maximum at $(1/2, 243/64)$. (Note that it is unnecessary to determine a simplified version of d^2y/dx^2 before inserting values of x.)

The argument $d^2y/dx^2 = 0$ at $x = 2 \Rightarrow C$ has a point of inflexion at $(2,0)$ is false. In fact, in the neighbourhood of this point, $y \approx 6(x-2)^4$, so that it is obvious that C has a minimum there.

The sketch of C must have correct overall shape, location and orientation, and also show correct forms at (0,0), (2,0) and at ∞.

(i) This sketch may be deduced from that of C. It has symmetry about the x - axis and no part of it appears in the region $-1 < x < 0$.

(ii) This sketch may also be deduced from that of C. It has symmetry about the y - axis and no part of it appears in the region $y < 0$.