STEP II, 2004, Q13 MS

Q13 If W_n pounds is the gain from draw n, then $E(W_{n+1}) = (b-r-n)/(b-n) \times 1 + r/(b-n) \times (-n)$ which is zero if $n = (b-r)/(r+1) = \xi$, say.

• W_{n+1} increases as n increases for $n < \xi$, and W_{n+1} decreases as n increases for $n > \xi$. Hence W_n maximum when $n = [\xi] + 1 = n_c$, say, so that optimal stopping n is n_c.

• For $r = 1$ and b even, $n_c = b/2$, in which case $P(\text{first } n_c - 1 \text{ draws are all white}) = (b - n_c + 1)/b = 1/2 + 1/b$.

Thus expected total reward $= (1/2 - 1/b) \times 0 + (1/2 + 1/b)\,[(b/2)/((b/2) + 1)] \times n_c = \ldots = b/4$ pounds.

• For $r = 1$ and b odd, $n_c = b/2 + 1/2$ so that now $P(\text{first } n_c - 1 \text{ draws are all white}) = 1/2 + 1/b$.

Hence expected total reward $= (1/2 + 1/2b) \times [(b/2 - 1/2)/(b + 1/2)] \times (b+1)/2 = \ldots = (b^2 - 1)/4b$ pounds.