

STEP II, 2004, Q12

- 12 Sketch the graph, for $x \geq 0$, of

$$y = kxe^{-ax^2},$$

where a and k are positive constants.

The random variable X has probability density function $f(x)$ given by

$$f(x) = \begin{cases} kxe^{-ax^2} & \text{for } 0 \leq x \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

Show that $k = \frac{2a}{1 - e^{-a}}$ and find the mode m in terms of a , distinguishing between the cases $a < \frac{1}{2}$ and $a > \frac{1}{2}$.

Find the median h in terms of a and show that $h > m$ if $a > -\ln(2e^{-1/2} - 1)$.

Show that $-\ln(2e^{-1/2} - 1) > \frac{1}{2}$. Show that, if $a > -\ln(2e^{-1/2} - 1)$, then

$$P(X > m \mid X < h) = \frac{2e^{-1/2} - e^{-a} - 1}{1 - e^{-a}}.$$



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