STEP II, 2011, Q11

11 The maximum power that can be developed by the engine of train A, of mass m, when travelling at speed v is $Pv^{3/2}$, where P is a constant. The maximum power that can be developed by the engine of train B, of mass $2m$, when travelling at speed v is $2Pv^{3/2}$. For both A and B resistance to motion is equal to kv, where k is a constant.

For $t \leqslant 0$, the engines are crawling along at very low equal speeds. At $t = 0$, both drivers switch on full power and at time t the speeds of A and B are v_A and v_B, respectively.

(i) Show that

$$v_A = \frac{P^2\left(1 - e^{-kt/2m}\right)^2}{k^2}$$

and write down the corresponding result for v_B.

(ii) Find v_A and v_B when $9v_A = 4v_B$. [You may find the substitution $v_A = u^2$ useful.]

(iii) Both engines are switched off when $9v_A = 4v_B$. Show that thereafter $k^2v_B^2 = 4P^2v_A$.