# STEP II, 2004, Q10 MS

*Q10* If the retardation of the particles when moving up the plane is $a_1\ ms^{-2}$, then $4g(\sqrt{3}/2)(1/5\sqrt{3})+2g = 4a_1 \Rightarrow a_1 = 6$, so that $P$ comes to rest after 1 second at $D$ where $AD = 3\,m$.

If the acceleration of $P$ down the slope is $a_2\ ms^{-2}$, then $-4g(\sqrt{3}/2)(1/5\sqrt{3}) + 2g = 4a_2 \Rightarrow a_2 = 4$.

Hence if $P$ and $Q$ meet at time $\tau$, then $3 - 2(\tau - 1)^2 = 6(\tau - T) - 3(\tau - T)^2$

$\Rightarrow \ldots \Rightarrow \tau^2 - (2 + 6T)\tau + 3T^2 + 6T + 1 = 0 \Rightarrow \ldots \Rightarrow \tau = 1 + (3 - \sqrt{6})T$.

Note that the condition $T < 1 + \sqrt{3/2}$ ensures that the collision takes place before $P$ returns to $A$.

(ii) A possible solution is first to show that $T = 1 + \sqrt{2/3} \Rightarrow \tau = 2$.

Hence as $v_P(2) = 4\ ms^{-2}$, $v_Q(2) = 2\sqrt{6}\ ms^{-2}$ then the total KE at $t = 2$ of $P$ and $Q = 80$ j.

Further, gain in PE at $t = 2$ since start of motion = 40 j so that energy lost due to friction = 144 - 80 - 40 = 24 j.

Alternatively, and more directly, the work done against friction up to the moment of collision = frictional force opposing motion of $P$ (or $Q$) $\times 6$ j $= 4 \times 6 = 24$ j.