# STEP II, 2003, Q7

7 Show that, if $n > 0$, then

$$\int_{e^{1/n}}^{\infty} \frac{\ln x}{x^{n+1}} \, \mathrm{d}x = \frac{2}{n^2 e} \, .$$

You may assume that $\dfrac{\ln x}{x} \to 0$ as $x \to \infty$.

Explain why, if $1 < a < b$, then

$$\int_{b}^{\infty} \frac{\ln x}{x^{n+1}} \, \mathrm{d}x < \int_{a}^{\infty} \frac{\ln x}{x^{n+1}} \, \mathrm{d}x \, .$$

Deduce that

$$\sum_{n=1}^{N} \frac{1}{n^2} < \frac{e}{2} \int_{e^{1/N}}^{\infty} \left( \frac{1 - x^{-N}}{x^2 - x} \right) \ln x \, \mathrm{d}x \, ,$$

where $N$ is any integer greater than 1.