

STEP II, 2003, Q7

7 Show that, if $n > 0$, then

$$\int_{e^{1/n}}^{\infty} \frac{\ln x}{x^{n+1}} dx = \frac{2}{n^2 e}.$$

You may assume that $\frac{\ln x}{x} \rightarrow 0$ as $x \rightarrow \infty$.

Explain why, if $1 < a < b$, then

$$\int_b^{\infty} \frac{\ln x}{x^{n+1}} dx < \int_a^{\infty} \frac{\ln x}{x^{n+1}} dx.$$

Deduce that

$$\sum_{n=1}^N \frac{1}{n^2} < \frac{e}{2} \int_{e^{1/N}}^{\infty} \left(\frac{1 - x^{-N}}{x^2 - x} \right) \ln x dx,$$

where N is any integer greater than 1.



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