STEP II, 2003, Q14

14 The probability of throwing a 6 with a biased die is p. It is known that p is equal to one or other of the numbers A and B where $0 < A < B < 1$. Accordingly the following statistical test of the hypothesis $H_0 : p = B$ against the alternative hypothesis $H_1 : p = A$ is performed.

The die is thrown repeatedly until a 6 is obtained. Then if X is the total number of throws, H_0 is accepted if $X \leqslant M$, where M is a given positive integer; otherwise H_1 is accepted. Let α be the probability that H_1 is accepted if H_0 is true, and let β be the probability that H_0 is accepted if H_1 is true.

Show that $\beta = 1 - \alpha^K$, where K is independent of M and is to be determined in terms of A and B. Sketch the graph of β against α.