STEP II, 2003, Q13

13 The random variable X takes the values $k = 1, 2, 3, \ldots$, and has probability distribution

$$\mathrm{P}(X = k) = A\frac{\lambda^k e^{-\lambda}}{k!},$$

where λ is a positive constant. Show that $A = (1 - \mathrm{e}^{-\lambda})^{-1}$. Find the mean μ in terms of λ and show that

$$\mathrm{Var}\,(X) = \mu(1 - \mu + \lambda) .$$

Deduce that $\lambda < \mu < 1 + \lambda$.

Use a normal approximation to find the value of $P(X = \lambda)$ in the case where $\lambda = 100$, giving your answer to 2 decimal places.