

## STEP II, 2003, Q12

- 12 The life of a certain species of elementary particles can be described as follows. Each particle has a life time of  $T$  seconds, after which it disintegrates into  $X$  particles of the same species, where  $X$  is a random variable with binomial distribution  $B(2, p)$ . A population of these particles starts with the creation of a single such particle at  $t = 0$ . Let  $X_n$  be the number of particles in existence in the time interval  $nT < t < (n + 1)T$ , where  $n = 1, 2, \dots$

Show that  $P(X_1 = 2 \text{ and } X_2 = 2) = 6p^4q^2$ , where  $q = 1 - p$ . Find the possible values of  $p$  if it is known that  $P(X_1 = 2|X_2 = 2) = 9/25$ .

Explain briefly why  $E(X_n) = 2pE(X_{n-1})$  and hence determine  $E(X_n)$  in terms of  $p$ . Show that for one of the values of  $p$  found above  $\lim_{n \rightarrow \infty} E(X_n) = 0$  and that for the other  $\lim_{n \rightarrow \infty} E(X_n) = +\infty$ .



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