STEP II, 2003, Q11

11 A particle P_1 is projected with speed V at an angle of elevation α $(> 45°)$, from a point in a horizontal plane. Find T_1, the flight time of P_1, in terms of α, V and g. Show that the time after projection at which the direction of motion of P_1 first makes an angle of $45°$ with the horizontal is $\frac{1}{2}(1 - \cot\alpha)T_1$.

A particle P_2 is projected under the same conditions. When the direction of the motion of P_2 first makes an angle of $45°$ with the horizontal, the speed of P_2 is instantaneously doubled. If T_2 is the total flight time of P_2, show that

$$\frac{2T_2}{T_1} = 1 + \cot\alpha + \sqrt{1 + 3\cot^2\alpha}\,.$$