STEP II, 2003, Q1

1 Consider the equations

$$\begin{aligned} ax - y - z &= 3\,, \\ 2ax - y - 3z &= 7\,, \\ 3ax - y - 5z &= b\,, \end{aligned}$$

where a and b are given constants.

(i) In the case $a = 0$, show that the equations have a solution if and only if $b = 11$.

(ii) In the case $a \neq 0$ and $b = 11$ show that the equations have a solution with $z = \lambda$ for any given number λ.

(iii) In the case $a = 2$ and $b = 11$ find the solution for which $x^2 + y^2 + z^2$ is least.

(iv) Find a value for a for which there is a solution such that $x > 10^6$ and $y^2 + z^2 < 1$.