STEP II, 2002, Q9

9 A particle is projected from a point O on a horizontal plane with speed V and at an angle of elevation α. The vertical plane in which the motion takes place is perpendicular to two vertical walls, both of height h, at distances a and b from O. Given that the particle just passes over the walls, find $\tan\alpha$ in terms of a, b and h and show that

$$\frac{2V^2}{g} = \frac{ab}{h} + \frac{(a+b)^2 h}{ab}\,.$$

The heights of the walls are now increased by the same small positive amount δh. A second particle is projected so that it just passes over both walls, and the new angle and speed of projection are $\alpha + \delta\alpha$ and $V + \delta V$, respectively. Show that

$$\sec^2\alpha\,\delta\alpha \approx \frac{a+b}{ab}\,\delta h\,,$$

and deduce that $\delta\alpha > 0$. Show also that δV is positive if $h > ab/(a+b)$ and negative if $h < ab/(a+b)$.