STEP II, 2002, Q8

8 Find y in terms of x, given that:

$$\text{for } x < 0\,, \quad \frac{\mathrm{d}y}{\mathrm{d}x} = -y \quad \text{and} \quad y = a \text{ when } x = -1\,;$$
$$\text{for } x > 0\,, \quad \frac{\mathrm{d}y}{\mathrm{d}x} = y \quad \text{and} \quad y = b \text{ when } x = 1\,.$$

Sketch a solution curve. Determine the condition on a and b for the solution curve to be continuous (that is, for there to be no 'jump' in the value of y) at $x = 0$.

Solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = |\,\mathrm{e}^x - 1|\,y$$

given that $y = \mathrm{e}^{\mathrm{e}}$ when $x = 1$ and that y is continuous at $x = 0$. Write down the following limits:

(i) $\lim_{x \to +\infty} y \exp(-\mathrm{e}^x)$; (ii) $\lim_{x \to -\infty} y\,\mathrm{e}^{-x}$.