

## STEP II, 2002, Q7

- 7 In 3-dimensional space, the lines  $m_1$  and  $m_2$  pass through the origin and have directions  $\mathbf{i} + \mathbf{j}$  and  $\mathbf{i} + \mathbf{k}$ , respectively. Find the directions of the two lines  $m_3$  and  $m_4$  that pass through the origin and make angles of  $\pi/4$  with both  $m_1$  and  $m_2$ . Find also the cosine of the acute angle between  $m_3$  and  $m_4$ .

The points  $A$  and  $B$  lie on  $m_1$  and  $m_2$  respectively, and are each at distance  $\lambda\sqrt{2}$  units from  $O$ . The points  $P$  and  $Q$  lie on  $m_3$  and  $m_4$  respectively, and are each at distance 1 unit from  $O$ . If all the coordinates (with respect to axes  $\mathbf{i}$ ,  $\mathbf{j}$  and  $\mathbf{k}$ ) of  $A$ ,  $B$ ,  $P$  and  $Q$  are non-negative, prove that:

- (i) there are only two values of  $\lambda$  for which  $AQ$  is perpendicular to  $BP$ ;
- (ii) there are no non-zero values of  $\lambda$  for which  $AQ$  and  $BP$  intersect.



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