

STEP II, 2002, Q5

5 The numbers x_n , where $n = 0, 1, 2, \dots$, satisfy

$$x_{n+1} = kx_n(1 - x_n).$$

- (i) Prove that, if $0 < k < 4$ and $0 < x_0 < 1$, then $0 < x_n < 1$ for all n .
- (ii) Given that $x_0 = x_1 = x_2 = \dots = a$, with $a \neq 0$ and $a \neq 1$, find k in terms of a .
- (iii) Given instead that $x_0 = x_2 = x_4 = \dots = a$, with $a \neq 0$ and $a \neq 1$, show that $ab^3 - b^2 + (1 - a) = 0$, where $b = k(1 - a)$. Given, in addition, that $x_1 \neq a$, find the possible values of k in terms of a .



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