

STEP II, 2002, Q3

- 3 The n th Fermat number, F_n , is defined by

$$F_n = 2^{2^n} + 1, \quad n = 0, 1, 2, \dots,$$

where 2^{2^n} means 2 raised to the power 2^n . Calculate F_0 , F_1 , F_2 and F_3 . Show that, for $k = 1$, $k = 2$ and $k = 3$,

$$F_0 F_1 \dots F_{k-1} = F_k - 2. \quad (*)$$

Prove, by induction, or otherwise, that (*) holds for all $k \geq 1$. Deduce that no two Fermat numbers have a common factor greater than 1.

Hence show that there are infinitely many prime numbers.



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