

STEP II, 2002, Q14

- 14 A densely populated circular island is divided into N concentric regions R_1, R_2, \dots, R_N , such that the inner and outer radii of R_n are $n - 1$ km and n km, respectively. The average number of road accidents that occur in any one day in R_n is $2 - n/N$, independently of the number of accidents in any other region.

Each day an observer selects a region at random, with a probability that is proportional to the area of the region, and records the number of road accidents, X , that occur in it. Show that, in the long term, the average number of recorded accidents per day will be

$$2 - \frac{1}{6} \left(1 + \frac{1}{N}\right) \left(4 - \frac{1}{N}\right).$$

[Note: $\sum_{n=1}^N n^2 = \frac{1}{6}N(N+1)(2N+1)$.]

Show also that

$$P(X = k) = \frac{e^{-2} N^{-k-2}}{k!} \sum_{n=1}^N (2n-1)(2N-n)^k e^{n/N}.$$

Suppose now that $N = 3$ and that, on a particular day, two accidents were recorded. Show that the probability that R_2 had been selected is

$$\frac{48}{48 + 45 e^{1/3} + 25 e^{-1/3}}.$$



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