

STEP II, 2002, Q13

- 13 Let $F(x)$ be the cumulative distribution function of a random variable X , which satisfies $F(a) = 0$ and $F(b) = 1$, where $a > 0$. Let

$$G(y) = \frac{F(y)}{2 - F(y)} .$$

Show that $G(a) = 0$, $G(b) = 1$ and that $G'(y) \geq 0$. Show also that

$$\frac{1}{2} \leq \frac{2}{(2 - F(y))^2} \leq 2 .$$

The random variable Y has cumulative distribution function $G(y)$. Show that

$$\frac{1}{2} E(X) \leq E(Y) \leq 2E(X) ,$$

and that

$$\text{Var}(Y) \leq 2 \text{Var}(X) + \frac{7}{4}(E(X))^2 .$$



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