STEP II, 2002, Q11

11 A rigid straight beam AB has length l and weight W. Its weight per unit length at a distance x from B is $\alpha W l^{-1}(x/l)^{\alpha-1}$, where α is a positive constant. Show that the centre of mass of the beam is at a distance $\alpha l/(\alpha+1)$ from B.

The beam is placed with the end A on a rough horizontal floor and the end B resting against a rough vertical wall. The beam is in a vertical plane at right angles to the plane of the wall and makes an angle of θ with the floor. The coefficient of friction between the floor and the beam is μ and the coefficient of friction between the wall and the beam is also μ. Show that, if the equilibrium is limiting at both A and B, then

$$\tan\theta = \frac{1-\alpha\mu^2}{(1+\alpha)\mu}\,.$$

Given that $\alpha = 3/2$ and given also that the beam slides for any $\theta < \pi/4$ find the greatest possible value of μ.