

## STEP II, 2001, Q8

8 The function  $f$  satisfies  $f(x + 1) = f(x)$  and  $f(x) > 0$  for all  $x$ .

(i) Give an example of such a function.

(ii) The function  $F$  satisfies

$$\frac{dF}{dx} = f(x)$$

and  $F(0) = 0$ . Show that  $F(n) = nF(1)$ , for any positive integer  $n$ .

(iii) Let  $y$  be the solution of the differential equation

$$\frac{dy}{dx} + f(x)y = 0$$

that satisfies  $y = 1$  when  $x = 0$ . Show that  $y(n) \rightarrow 0$  as  $n \rightarrow \infty$ , where  $n = 1, 2, 3, \dots$



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