STEP II, 2001 Q5

5 The curve C_1 passes through the origin in the x–y plane and its gradient is given by

$$\frac{\mathrm{d}y}{\mathrm{d}x} = x(1-x^2)e^{-x^2}.$$

Show that C_1 has a minimum point at the origin and a maximum point at $\left(1, \frac{1}{2}e^{-1}\right)$. Find the coordinates of the other stationary point. Give a rough sketch of C_1.

The curve C_2 passes through the origin and its gradient is given by

$$\frac{\mathrm{d}y}{\mathrm{d}x} = x(1-x^2)e^{-x^3}.$$

Show that C_2 has a minimum point at the origin and a maximum point at $(1, k)$, where $k > \frac{1}{2}e^{-1}$. (You need not find k.)