STEP II, 2001, Q4

4 Let

$$\mathrm{f}(x) = P\sin x + Q\sin 2x + R\sin 3x\ .$$

Show that if $Q^2 < 4R(P-R)$, then the only values of x for which $\mathrm{f}(x) = 0$ are given by $x = m\pi$, where m is an integer.
[You may assume that $\sin 3x = \sin x(4\cos^2 x - 1)$.]

Now let

$$\mathrm{g}(x) = \sin 2nx + \sin 4nx - \sin 6nx,$$

where n is a positive integer and $0 < x < \pi/2$. Find an expression for the largest root of the equation $\mathrm{g}(x) = 0$, distinguishing between the cases where n is even and n is odd.