

## STEP II, 2001, Q3

- 3** The cuboid  $ABCDEFGH$  is such  $AE, BF, CG, DH$  are perpendicular to the opposite faces  $ABCD$  and  $EFGH$ , and  $AB = 2, BC = 1, AE = \lambda$ . Show that if  $\alpha$  is the acute angle between the diagonals  $AG$  and  $BH$  then

$$\cos \alpha = \left| \frac{3 - \lambda^2}{5 + \lambda^2} \right|$$

Let  $R$  be the ratio of the volume of the cuboid to its surface area. Show that  $R < 1/3$  for all possible values of  $\lambda$ .

Prove that, if  $R \geq 1/4$ , then  $\alpha \leq \arccos(1/9)$ .



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