STEP II, 2001, Q3

3 The cuboid $ABCDEFGH$ is such AE, BF, CG, DH are perpendicular to the opposite faces $ABCD$ and $EFGH$, and $AB = 2, BC = 1, AE = \lambda$. Show that if α is the acute angle between the diagonals AG and BH then

$$\cos\alpha = \left|\frac{3-\lambda^2}{5+\lambda^2}\right|$$

Let R be the ratio of the volume of the cuboid to its surface area. Show that $R < 1/3$ for all possible values of λ.

Prove that, if $R \geqslant 1/4$, then $\alpha \leqslant \arccos(1/9)$.