

## STEP II, 2000, Q6

6 Show that

$$\sin \theta = \frac{2t}{1+t^2}, \quad \cos \theta = \frac{1-t^2}{1+t^2}, \quad \frac{1+\cos \theta}{\sin \theta} = \tan(\pi/2 - \theta/2),$$

where  $t = \tan(\theta/2)$ .

Use the substitution  $t = \tan(\theta/2)$  to show that, for  $0 < \alpha < \pi/2$ ,

$$\int_0^{\pi/2} \frac{1}{1 + \cos \alpha \sin \theta} d\theta = \frac{\alpha}{\sin \alpha},$$

and deduce a similar result for

$$\int_0^{\pi/2} \frac{1}{1 + \sin \alpha \cos \theta} d\theta.$$



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