

STEP II, 2000, Q5

- 5 It is required to approximate a given function $f(x)$, over the interval $0 \leq x \leq 1$, by the linear function λx , where λ is chosen to minimise

$$\int_0^1 (f(x) - \lambda x)^2 dx.$$

Show that

$$\lambda = 3 \int_0^1 xf(x) dx.$$

The residual error, R , of this approximation process is such that

$$R^2 = \int_0^1 (f(x) - \lambda x)^2 dx.$$

Show that

$$R^2 = \int_0^1 (f(x))^2 dx - \frac{1}{3}\lambda^2.$$

Given now that $f(x) = \sin(\pi x/n)$, show that (i) for large n , $\lambda \approx \pi/n$ and (ii) $\lim_{n \rightarrow \infty} R = 0$.

Explain why, prior to any calculation, these results are to be expected.

[You may assume that, when θ is small, $\sin \theta \approx \theta - \theta^3/6$ and $\cos \theta \approx 1 - \theta^2/2$.]



NextStepMaths.com

To view mark schemes, fully worked solutions and examiner's comments, and for more details about tutoring and other services offered, go to NextStepMaths.com