



STEP II, 2000, Q4

4 Prove that

$$(\cos \theta + i \sin \theta)(\cos \phi + i \sin \phi) = \cos(\theta + \phi) + i \sin(\theta + \phi)$$

and that, for every positive integer n ,

$$(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta.$$

By considering $(5 - i)^2(1 + i)$, or otherwise, prove that

$$\arctan(7/17) + 2 \arctan(1/5) = \pi/4.$$

Prove also that

$$3 \arctan(1/4) + \arctan(1/20) + \arctan(1/1985) = \pi/4.$$

[Note that $\arctan \theta$ is another notation for $\tan^{-1} \theta$.]



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