

STEP II, 2000, Q14

- 14 The random variables $X_1, X_2, \dots, X_{2n+1}$ are independently and uniformly distributed on the interval $0 \leq x \leq 1$. The random variable Y is defined to be the median of $X_1, X_2, \dots, X_{2n+1}$. Given that the probability density function of Y is $g(y)$, where

$$g(y) = \begin{cases} ky^n(1-y)^n & \text{if } 0 \leq y \leq 1 \\ 0 & \text{otherwise,} \end{cases}$$

use the result

$$\int_0^1 y^r(1-y)^s dy = \frac{r!s!}{(r+s+1)!}$$

to show that $k = (2n+1)!/(n!)^2$, and evaluate $E(Y)$ and $\text{Var}(Y)$. Hence show that, for any given positive number d , the inequality

$$P(|Y - 1/2| < d/\sqrt{n}) < P(|\bar{X} - 1/2| < d/\sqrt{n})$$

holds provided n is large enough, where \bar{X} is the mean of $X_1, X_2, \dots, X_{2n+1}$.

[You may assume that Y and \bar{X} are normally distributed for large n .]



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