STEP II, 2000, Q13

13 A group of biologists attempts to estimate the magnitude, N, of an island population of voles (*Microtus agrestis*). Accordingly, the biologists capture a random sample of 200 voles, mark them and release them. A second random sample of 200 voles is then taken of which 11 are found to be marked. Show that the probability, p_N, of this occurrence is given by

$$p_N = k\frac{\big((N-200)!\big)^2}{N!(N-389)!},$$

where k is independent of N.

The biologists then estimate N by calculating the value of N for which p_N is a maximum. Find this estimate.

All unmarked voles in the second sample are marked and then the entire sample is released. Subsequently a third random sample of 200 voles is taken. Write down the probability that this sample contains exactly j marked voles, leaving your answer in terms of binomial coefficients.

Deduce that

$$\sum_{j=0}^{200}\binom{389}{j}\binom{3247}{200-j} = \binom{3636}{200}.$$