

## **STEP II, 2000, Q13**

- 13** A group of biologists attempts to estimate the magnitude,  $N$ , of an island population of voles (*Microtus agrestis*). Accordingly, the biologists capture a random sample of 200 voles, mark them and release them. A second random sample of 200 voles is then taken of which 11 are found to be marked. Show that the probability,  $p_N$ , of this occurrence is given by

$$p_N = k \frac{((N - 200)!)^2}{N!(N - 389)!},$$

where  $k$  is independent of  $N$ .

The biologists then estimate  $N$  by calculating the value of  $N$  for which  $p_N$  is a maximum. Find this estimate.

All unmarked voles in the second sample are marked and then the entire sample is released. Subsequently a third random sample of 200 voles is taken. Write down the probability that this sample contains exactly  $j$  marked voles, leaving your answer in terms of binomial coefficients.

Deduce that

$$\sum_{j=0}^{200} \binom{389}{j} \binom{3247}{200-j} = \binom{3636}{200}.$$



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