



## STEP II, 2000, Q1

- 1 A number of the form  $1/N$ , where  $N$  is an integer greater than 1, is called a *unit fraction*.

Noting that

$$\frac{1}{2} = \frac{1}{3} + \frac{1}{6} \quad \text{and} \quad \frac{1}{3} = \frac{1}{4} + \frac{1}{12},$$

guess a general result of the form

$$\frac{1}{N} = \frac{1}{a} + \frac{1}{b} \quad (*)$$

and hence prove that any unit fraction can be expressed as the sum of two distinct unit fractions.

By writing (\*) in the form

$$(a - N)(b - N) = N^2$$

and by considering the factors of  $N^2$ , show that if  $N$  is prime, then there is only one way of expressing  $1/N$  as the sum of two distinct unit fractions.

Prove similarly that any fraction of the form  $2/N$ , where  $N$  is prime number greater than 2, can be expressed uniquely as the sum of two distinct unit fractions.



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