STEP II, 1999, Q9

9 In the Z–universe, a star of mass M suddenly blows up, and the fragments, with various initial speeds, start to move away from the centre of mass G which may be regarded as a fixed point. In the subsequent motion the acceleration of each fragment is directed towards G. Moreover, in accordance with the laws of physics of the Z–universe, there are positive constants k_1, k_2 and R such that when a fragment is at a distance x from G, the magnitude of its acceleration is k_1x^3 if $x < R$ and is k_2x^{-4} if $x \geqslant R$. The initial speed of a fragment is denoted by u.

(i) For $x < R$, write down a differential equation for the speed v, and hence determine v in terms of u, k_1 and x for $x < R$.

(ii) Show that if $u < a$, where $2a^2 = k_1R^4$, then the fragment does not reach a distance R from G.

(iii) Show that if $u \geqslant b$, where $6b^2 = 3k_1R^4 + 4k_2/R^3$, then from the moment of the explosion the fragment is always moving away from G.

(iv) If $a < u < b$, determine in terms of k_2, b and u the maximum distance from G attained by the fragment.