STEP II, 1999, Q8

8 Prove that

$$\sum_{k=0}^{n} \sin k\theta = \frac{\cos \frac{1}{2}\theta - \cos(n+\frac{1}{2})\theta}{2\sin\frac{1}{2}\theta}\,. \qquad (*)$$

(i) Deduce that, when n is large,

$$\sum_{k=0}^{n} \sin\left(\frac{k\pi}{n}\right) \approx \frac{2n}{\pi}\,.$$

(ii) By differentiating $(*)$ with respect to θ, or otherwise, show that, when n is large,

$$\sum_{k=0}^{n} k\sin^2\left(\frac{k\pi}{2n}\right) \approx \left(\frac{1}{4}+\frac{1}{\pi^2}\right)n^2\,.$$

[The approximations, valid for small θ, $\sin\theta \approx \theta$ and $\cos\theta \approx 1 - \frac{1}{2}\theta^2$ may be assumed.]

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