

STEP II, 1999, Q4

- 4 By considering the expansions in powers of x of both sides of the identity

$$(1+x)^n(1+x)^n \equiv (1+x)^{2n},$$

show that

$$\sum_{s=0}^n \binom{n}{s}^2 = \binom{2n}{n},$$

where $\binom{n}{s} = \frac{n!}{s!(n-s)!}$.

By considering similar identities, or otherwise, show also that:

- (i) if n is an even integer, then

$$\sum_{s=0}^n (-1)^s \binom{n}{s}^2 = (-1)^{n/2} \binom{n}{n/2};$$

(ii) $\sum_{t=1}^n 2t \binom{n}{t}^2 = n \binom{2n}{n}$.



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