

STEP II, 1999, Q3

3 Let

$$S_n(x) = e^{x^3} \frac{d^n}{dx^n} (e^{-x^3}).$$

Show that $S_2(x) = 9x^4 - 6x$ and find $S_3(x)$.

Prove by induction on n that $S_n(x)$ is a polynomial. By means of your induction argument, determine the order of this polynomial and the coefficient of the highest power of x .

Show also that if $\frac{dS_n}{dx} = 0$ for some value a of x , then $S_n(a)S_{n+1}(a) \leq 0$.



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