

## STEP II, 1999, Q2

2 Consider the quadratic equation

$$nx^2 + 2x\sqrt{(pn^2 + q)} + rn + s = 0, \quad (*)$$

where  $p > 0$ ,  $p \neq r$  and  $n = 1, 2, 3, \dots$

- (i) For the case where  $p = 3$ ,  $q = 50$ ,  $r = 2$ ,  $s = 15$ , find the set of values of  $n$  for which equation (\*) has no real roots.
- (ii) Prove that if  $p < r$  and  $4q(p - r) > s^2$ , then (\*) has no real roots for any value of  $n$ .
- (iii) If  $n = 1$ ,  $p - r = 1$  and  $q = s^2/8$ , show that (\*) has real roots if, and only if,  $s \leq 4 - 2\sqrt{2}$  or  $s \geq 4 + 2\sqrt{2}$ .



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